Machine Learning 1 – Exercise 1

Machine Learning for Computer Vision
TU Dresden

Solutions to any part of any of the exercises below will be accepted as separate entries of the thread entitled **Exercise 1: Solution** of the lecture forum[1] until **Nov 19th, 18:00**. The solutions will not be graded. Instead, at the end of this term, the most highly voted solution of all will be awarded with a book prize.

1 Deciding with disjunctive normal forms (DNFs)

a) Let $V = \{0, 1, 2, 3\}$. State the V-variate DNF defined by $\theta = \{(\emptyset, \{0\}),\ (\{0\}, \{3\}),\ (\{0, 3\}, \{1, 2\})\}$, its length and its depth.

b) State two distinct DNFs such that the function defined by these DNFs equals the function g defined in Tab. 1 below.

c) How many distinct DNFs in $n = |V|$ variables exist?

d) Prove the following universality lemma: For any finite, non-empty set V and any $f\colon \{0, 1\}^V \to \{0, 1\}$, there exists a V-variate DNF defining f.

2 Deciding with binary decision trees (BDTs)

a) Construct two distinct BDTs such that the function defined by these BDTs equals the function g defined in Tab. 1.

b) Let V be a finite, non-empty set. Define an algorithm that takes any disjoint sets $A, B \subseteq \{0, 1\}^V$ as the input and outputs a V-variate, $\{0, 1\}$-valued BDT θ such that the function f_θ defined by this BDT has the properties $f_\theta(A) = 0$ and $f_\theta(B) = 1$.

c) Prove the correctness of your algorithm.

d) Prove the following universality lemma: For any finite, non-empty set V and any $f\colon \{0, 1\}^V \to \{0, 1\}$, there exists a V-variate, $\{0, 1\}$-valued BDT defining f.

[1] `https://bildungsportal.sachsen.de/opal/auth/RepositoryEntry/26617479170/CourseNode/102502724177602`

Table 1: Defined by the value table below is a Boolean function $g\colon \{0,1\}^V \to \{0,1\}$ with $V = \{0,1,2\}$.

x_0	x_1	x_2	$g(x)$
0	0	0	0
1	0	0	1
0	1	0	0
1	1	0	1
0	0	1	1
1	0	1	0
0	1	1	1
1	1	1	0